

NAG Toolbox for MATLAB

f08pn

1 Purpose

f08pn computes the eigenvalues, the Schur form T , and, optionally, the matrix of Schur vectors Z for an n by n complex nonsymmetric matrix A .

2 Syntax

```
[a, sdim, w, vs, info] = f08pn(jobvs, sort, select, a, 'n', n)
```

3 Description

The Schur factorization of A is given by

$$A = ZTZ^H,$$

where Z is orthogonal, the matrix of Schur vectors, and T is upper triangular.

Optionally, f08pn also orders the eigenvalues on the diagonal of the Schur form so that selected eigenvalues are at the top left. The leading columns of Z form an orthonormal basis for the invariant subspace corresponding to the selected eigenvalues.

A complex matrix is in Schur form if it is upper triangular.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobvs** – string

If **jobvs** = 'N', Schur vectors are not computed.

If **jobvs** = 'V', Schur vectors are computed.

Constraint: **jobvs** = 'N' or 'V'.

2: **sort** – string

Specifies whether or not to order the eigenvalues on the diagonal of the Schur form.

sort = 'N'

Eigenvalues are not ordered.

sort = 'S'

Eigenvalues are ordered (see user-supplied logical function **select**).

Constraint: **sort** = 'N' or 'S'.

3: **select** – string containing name of m-file

If **sort** = 'S', **select** is used to select eigenvalues to sort to the top left of the Schur form.

If **sort** = 'N', **select** is not referenced and f08pn may be called with the string 'f08pnz'.

Its specification is:

```
[result] = select(w)
```

Input Parameters

1: **w** – complex scalar

The eigenvalue $w(j)$ is selected if **select**($w(j)$) is **true**.

Output Parameters

1: **result** – logical scalar

The result of the function.

4: **a(lda,*)** – complex array

The first dimension of the array **a** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

The n by n matrix A .

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix A .

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldvs, work, lwork, rwork, bwork

5.4 Output Parameters

1: **a(lda,*)** – complex array

The first dimension of the array **a** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

a contains its Schur form T .

2: **sdim** – int32 scalar

If **sort** = 'N', **sdim** = 0.

If **sort** = 'S', **sdim** = number of eigenvalues for which user-supplied logical function **select** is **true**.

3: **w(*)** – complex array

Note: the dimension of the array **w** must be at least $\max(1, n)$.

Contains the computed eigenvalues, in the same order that they appear on the diagonal of the output Schur form T .

4: **vs(ldvs,*)** – **complex array**

The first dimension, **ldvs**, of the array **vs** must satisfy

if **jobvs** = 'V', **ldvs** $\geq \max(1, n)$;
ldvs ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, n)$ if **jobvs** = 'V', and at least 1 otherwise

If **jobvs** = 'V', **vs** contains the unitary matrix Z of Schur vectors.

If **jobvs** = 'N', **vs** is not referenced.

5: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobvs**, 2: **sort**, 3: **select**, 4: **n**, 5: **a**, 6: **lda**, 7: **sdim**, 8: **w**, 9: **vs**, 10: **ldvs**, 11: **work**, 12: **lwork**,
 13: **rwork**, 14: **bwork**, 15: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info = 1 to N

If **info** = i and $i \leq n$, the QR algorithm failed to compute all the eigenvalues

info = $N + 1$

The eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned).

info = $N + 2$

After reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy **select** = **true**. This could also be caused by underflow due to scaling.

7 Accuracy

The computed Schur factorization satisfies

$$A + E = ZTZ^H,$$

where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. See Section 4.8 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3 .

The real analogue of this function is f08pa.

9 Example

f08pn_select.m

```

jobvs = 'Vectors (Schur)';
sort = 'No sort';
a = [complex(-3.97, -5.04), complex(-4.11, +3.7), complex(-0.34, +1.01),
      complex(1.29, -0.86);
      complex(0.34, -1.5), complex(1.52, -0.43), complex(1.88, -5.38),
      complex(3.36, +0.65);
      complex(3.31, -3.85), complex(2.5, +3.45), complex(0.88, -1.08),
      complex(0.64, -1.48);
      complex(-1.1, +0.82), complex(1.81, -1.59), complex(3.25, +1.33),
      complex(1.57, -3.44)];
[aOut, sdim, w, vs, info] = f08pn(jobvs, sort, 'f08pn_select', a)

aOut =
   -6.0004 - 6.9998i   -0.3656 + 0.3637i    0.4761 - 0.1946i   -0.7237 +
0.5589i
      0                -5.0000 + 2.0060i    0.4981 - 0.5232i   -0.1637 +
0.2071i
      0                  0                7.9982 - 0.9964i    0.8487 -
0.6651i
      0                  0                  0                3.0023 -
3.9998i
sdim =
      0

w =
   -6.0004 - 6.9998i
   -5.0000 + 2.0060i
    7.9982 - 0.9964i
    3.0023 - 3.9998i

vs =
   -0.5312 - 0.6581i    0.3018 - 0.2402i   -0.0935 - 0.2736i   -0.1355 +
0.1844i
    0.2474 - 0.1769i    0.1759 - 0.5473i   -0.4015 + 0.6010i   -0.0027 -
0.2337i
    0.1874 - 0.2637i   -0.6002 + 0.1953i   -0.5752 - 0.0389i   -0.1458 +
0.3787i
   -0.1909 + 0.2262i    0.2027 + 0.2852i   -0.1537 + 0.1951i   -0.8459 -
0.1130i
info =
      0

```